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Quasistatic Optimal Actuator Placement with Minimum Worst Case Distortion Criterion

Saki Hakim* and Maurice Bernard Fuchs†
Tel-Aviv University, 69978 Ramat Aviv, Israel

The problem of placing actuators optimally in truss structures for quasistatic shape control is considered. Optimal actuator placement in large truss structures is a formidable task due to the combinatorial nature of the problem. In this paper we use and compare the performances of several heuristic search techniques to determine their effectiveness in solving this design problem. The selected methods are simulated annealing, exhaustive single-point substitution, and single location iterative minimization, a sequential minimization technique presented herein. The optimality criterion is the worst case distortion after correction. This worst case value of a performance measure is obtained through a constrained min-max procedure. An ideal actuator concept is employed to determine limits on the optimal criterion value, which is then used also to evaluate the number of actuators needed for given levels of correction. The procedures just discussed are performed on representative trusses. Results show that heuristics can handle the optimization problem since the number of function calls grows linearly with the size of the problem and not in a combinatorial fashion, typical in such cases. The different algorithms exhibit similar performances; however, exhaustive single-point substitution requires a significantly higher computational effort. The applicability of the lower limit is investigated, and in most cases there is good agreement with the optimization results.

Introduction

SPACE missions have ever more demanding requirements from space structures. They must be large, light, and easy to assemble. A natural choice answering these demands are trusses.[1] The shape control problem arises when the structure has to support devices requiring a high degree of accuracy, such as antennas or multilens telescopes. Since structures of that type are large and flexible, they are very sensitive to environmental effects. For example, periodic changes of the temperature field often result in unacceptable distortions. Consequently, active control systems seem unavoidable.[2] These systems consist of sensors that estimate the distortions[3] and actuators that are there to reduce the deformations.

The actuators are often embedded in selected axial elements[4] and control the structure by extending or contracting when responding to a command from the control system. The number of actuators is usually bounded by technological and economical factors. Consequently, the distortions cannot be fully annulled, and as a result, an adequate positioning of the actuators is imperative. Thus the objective is to find actuator locations that yield optimum performance. To do so, all of the possibilities should be explored. This can be done in small structures by means of exhaustive search. In large structures exhaustive search is not feasible because of the combinatorial nature of the problem.

There exists a gamut of such combinatorial problems in all walks of engineering and applied science. They belong to the nonpolynomial complete (NP-complete) class of problems[5] for which there is no proven algorithm that can provide the solution in polynomial, e.g., tractable, time. Such problems are more often than not addressed by approximation algorithms or heuristics. Although these techniques cover only a fraction of the configuration space, they often navigate near the solution and sometimes at the solution. In fact, in these methods the optimality of the solution is traded with its computation time. The family of techniques we consider herein are part of iterative improvement algorithms and include exhaustive single-point substitution (ESPS),[6] single location iterative minimization (SLIM), and simulated annealing (SA).[7,8] SLIM is a sequential minimization technique proposed herein that is closely related to ESPS. SA is highly dependent on its cooling schedule, and to circumvent that dependency the values of the parameters governing the SA and its cooling schedule are determined by the number of available locations, the number of actuators, and the lower limit of the criterion value. Therefore the specific SA algorithm developed herein is adaptive, that is, problem-size independent, and does not require additional, user-supplied, tuning parameters to accommodate for the size of the problem.

SA differs from ESPS and SLIM in two major aspects. SA proceeds slowly from one configuration to close ones, generated randomly. It usually selects better configurations but allowance is made for selecting worse designs in some cases. The algorithm can thus escape from local minima in its quest for the global minimum. ESPS and SLIM are mostly deterministic. From a given configuration they search for a better configuration that differs from the current design by one actuator and proceed until no further improvement; that is, only improving solutions are accepted. Randomness is introduced at the very start of the process by determining an initial random configuration. The aim of this paper is to ascertain whether the solutions obtained with iterative improvement methods are acceptable and whether the computation effort is reasonable.

To test the effectiveness of a configuration of actuators, several optimality criteria have been used in literature. A simple method is to assume that the distortion is a known single mode deformation.[2,9] In this case the optimality criterion is the root mean square (rms) of the corrected shape. However the configuration thus obtained is designed for that disturbance only, and other disturbances may yield different solutions. In the case of many disturbance sources one could use a criterion that is based only on the positions of the actuators, such as in Das et al.[1] This approach is to make $\boldsymbol{C}$, the influence matrix of the actuators on the controlled degrees of freedom (CDOF), large in some sense. The rationale of this approach is that since the disturbance is unknown and complete correction is not feasible, the best we can hope for is to use small control forces. Criteria such as $\max\|\boldsymbol{C}\|$ or $\max \det(\boldsymbol{C}^T\boldsymbol{C})$ have been used.[1] A different approach[1,10] dealing with arbitrary disturbances is to assume a Gaussian distribution of the disturbance. In this case the optimal placement is the one yielding minimum expected value of the corrected shape. A nonstatistical approach is presented in Ref. 11, where the criterion is the worst case ratio of the controlled to uncontrolled rms distortion, and the distortion is assumed to belong to a family of functions that do not vary much along the structure.

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*Doctoral Student, Department of Solid Mechanics, Materials, and Structures, Iby and Aladar Fleischman Faculty of Engineering.

†Associate Professor, Department of Solid Mechanics, Materials, and Structures, Iby and Aladar Fleischman Faculty of Engineering.

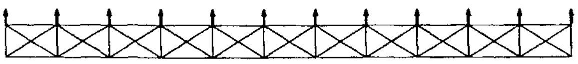

Fig. 1 Controlled degrees of freedom.

In the present paper we use a recently introduced criterion of worst case distortion.[12] We start from unknown disturbances that affect the structure, and we assume that the disturbance influence matrix $\boldsymbol{D}$ can be established. A performance measure is defined, which employs both the control and disturbance influence matrices. The criterion is the worst case value of that performance measure. To obtain the worst case performance measure for a given configuration, a constrained min-max procedure is defined. The control forces minimize the performance measure for disturbances that maximize it, with a constraint on the disturbance magnitude.

A special section is dedicated to reducing the computational effort required for calculating the criterion. Noteworthy is the derivation of a lower limit on the criterion value that enables one to calculate the best performance one can expect from a given number of actuators without any search method. This limit allows one to determine a priori the number of required actuators, and it also serves as a basis to assess the results of the heuristic search methods. For the development of the limit, a concept of ideal actuators, proposed by Haftka,[13] was used. The search techniques in conjunction with the worst case optimality criterion are applied to a two-dimensional beam truss and a three-dimensional hexagonal antenna truss. Their relative performances are eventually compared and discussed.

Shape Control

The shape control problem arises, for instance, when the structure has to support devices requiring a high degree of accuracy such as antenna or multilens telescopes. The precision requirements are on a subset of the structure displacements, the CDOF, which are situated where the devices are attached to the structure. Consider for instance the two-dimensional truss in Fig. 1, and let us assume that the precision demands are that the upper chord of the truss should remain a straight line. The CDOF are here the 12 vertical displacements of the upper chord nodes (arrows in the figure). This vector is a subset of the 48 components of the nodal displacements vector.

Because of some external agent, the truss will experience nonzero displacements, in particular at these CDOF. In the following we will term the external agent that affects the structure disturbances. These disturbances will cause deformations or strains in the structure, and the resulting displacements at the CDOF will be called distortions. Assuming that we select N_p displacements for the CDOF set, we represent the distortions at the CDOF by

$$\boldsymbol{v}_d = \boldsymbol{D}\boldsymbol{d} \tag{1}$$

where $\boldsymbol{d}$ is a disturbance vector of size N_d. The disturbance can be of any type such as temperature gradients, element size errors, and so forth, and N_d depends on the number of disturbance sources and on the way they are parameterized. The distortions $\boldsymbol{v}_d$ are the displacements at the CDOF due to these disturbances, and $\boldsymbol{D}$ is the corresponding influence matrix. Note that $\boldsymbol{v}$ is the displacements of the CDOF and not of all of the nodes.

Control is applied through active elements that can be instructed to modify their length by a desired amount. In truss structures it is very convenient to embed such actuators in some of the truss elements that thus become active elements. The influence relation of the active elements on the CDOF is

$$\boldsymbol{v}_c = \boldsymbol{C}\boldsymbol{c} \tag{2}$$

where $\boldsymbol{v}_c$ is the displacements at the CDOF due to induced elongation $\boldsymbol{c}$ at the active bars, which are used to correct the distortion $\boldsymbol{v}_d$. Assuming N_c elements with actuators, the control influence matrix $\boldsymbol{C}$ is of size $N_p \times N_c$. The control vector $\boldsymbol{c}$ should be viewed as generalized forces and can be actual forces, elongations, temperatures (thermally induced elongations), or voltages (in the case of piezoelectric actuators).

The influence matrices $\boldsymbol{D}$ and $\boldsymbol{C}$ are obtained with standard structural analysis for floating structures, where only the CDOF are constrained to have no rigid-body modes. It is understood that the control system is divided into a rigid-body controller, which rules the overall position and orientation of the structure, and a shape control system, which alleviates the distortions of the CDOF. By constraining the CDOF to have no rigid-body modes, the shape controller is thus decoupled from the position/orientation control. We define the error $\boldsymbol{e}$ as the difference between the distortion and the correction at the CDOF:

$$\boldsymbol{e} = \boldsymbol{v}_d - \boldsymbol{v}_c \tag{3}$$

For precision control it is customary to minimize the rms of the error, that is, to minimize $\boldsymbol{e}^T\boldsymbol{e}$. This demand can be generalized to include the cost of the control effort, which leads to a performance measure similar to the one used in dynamic optimal control for linear quadratic regulators

$$J \equiv \boldsymbol{e}^T\boldsymbol{P}\boldsymbol{e} + \boldsymbol{c}^T\boldsymbol{W}\boldsymbol{c} \tag{4}$$

where $\boldsymbol{P}$ is a symmetric positive definite weighting matrix of the error and $\boldsymbol{W}$ is a symmetric positive semidefinite weighting matrix of the control effort. The first term on the right-hand side (RHS) of Eq. (4) is a measure of the error, after correction. The second term should be construed as a penalty term that is introduced to limit the control forces. The weighting matrices define a proper mix between the goal of minimizing the rms of the error and also the cost of the control. For every configuration of actuators, when the disturbance $\boldsymbol{d}$ and the control forces $\boldsymbol{c}$ are given, the value of J can be calculated.

This performance measure will be used to define the optimal control law and to compare the performances of different configurations. Note that a control law is the relation between the control forces $\boldsymbol{c}$ and the disturbance $\boldsymbol{d}$ to correct the error $\boldsymbol{e}$. When there are many disturbance sources, every combination of disturbances yields a different distortion. The difficulty resides in selecting a representative disturbance. A solution for this problem was proposed in Hakim and Fuchs.[12] For every candidate configuration of actuators one selects the disturbance that causes the worst value of J. Consequently, for every tested configuration of actuators, we solve a min-max problem. To take into account worst case situations, we maximize J with respect to the disturbances $\boldsymbol{d}$. On the other hand, for best control, we seek $\boldsymbol{c}$ (control forces) that minimizes J. Since in reality the disturbance has finite energy, we introduce a constraint on the magnitude of $\boldsymbol{d}$ that for ease of manipulation is taken to be 1, and the columns of $\boldsymbol{D}$ are scaled accordingly. We can now formulate the min-max problem that will yield the optimality criterion J^*, subject to $\boldsymbol{d}^T\boldsymbol{d} - 1 = 0$,

$$J^* \equiv \min_{c}\max_{d} J \tag{5}$$

To solve the min-max problem we form the Lagrangian:

$$L = J - \lambda(\boldsymbol{d}^T\boldsymbol{d} - 1) \tag{6}$$

where λ is a Lagrangian multiplier. Let $\boldsymbol{d}^*$, $\boldsymbol{c}^*$, and λ^* be the values that extremize the Lagrangian. The necessary conditions for extremum L are

$$\frac{\partial L}{\partial \boldsymbol{c}} = 2\frac{\partial \boldsymbol{e}}{\partial \boldsymbol{c}}\boldsymbol{P}\boldsymbol{e} + 2\boldsymbol{W}\boldsymbol{c} = -2\boldsymbol{C}^T\boldsymbol{P}(\boldsymbol{D}\boldsymbol{d} - \boldsymbol{C}\boldsymbol{c}) + 2\boldsymbol{W}\boldsymbol{c} = \boldsymbol{0} \tag{7}$$

$$\frac{\partial L}{\partial \boldsymbol{d}} = 2\frac{\partial \boldsymbol{e}}{\partial \boldsymbol{d}}\boldsymbol{P}\boldsymbol{e} - 2\lambda\boldsymbol{d} = 2\boldsymbol{D}^T\boldsymbol{P}(\boldsymbol{D}\boldsymbol{d} - \boldsymbol{C}\boldsymbol{c}) - 2\lambda\boldsymbol{d} = \boldsymbol{0} \tag{8}$$

$$\frac{\partial L}{\partial \lambda} = \boldsymbol{d}^T\boldsymbol{d} - 1 = 0 \tag{9}$$

We can solve Eq. (8) for $\boldsymbol{c}^*$:

$$\boldsymbol{c}^* = [(\boldsymbol{C}^T\boldsymbol{P}\boldsymbol{C} + \boldsymbol{W})^{-1}\boldsymbol{C}^T\boldsymbol{P}\boldsymbol{D}]\boldsymbol{d} \tag{10}$$

thus establishing the control law. The optimal control forces $\boldsymbol{c}^*$ are a linear function of the disturbances, and hence the control law is independent of the disturbance.

Substituting the control law (10) into Eq. (8) leads to an eigenvalue problem,

$$(A - \lambda I)d = 0 \tag{11}$$

with

$$A = D^T[P - PC(C^T PC + W)^{-1}C^T P]D \tag{12}$$

the solution of which are eigenpairs $\{\lambda_i, d_i\}$ of A. It is shown in Ref. 12 that the optimal value of J, J^*, is the largest eigenvalue of A, λ^*, and the eigenvector d^* yields the worst case distortion v_d^*, that is, $v_d^* = Dd^*$. The term J^* is the worst case value of the performance measure for that configuration. For each configuration d^* is different and in each configuration $d \neq d^*$ yields $J \leq J^*$, provided we use the optimal control law (10).

To compare the performance of different configurations we calculate J^* for each set of actuator locations and the one with min J^* is the optimal solution to the problem. Since J^* will be calculated many times during the optimization process, we show in the next section how the computational effort can be reduced in the cases where the number of disturbance sources is larger than the number of CDOF.

Computing J^*

To compute the value of J^* for a candidate set of actuator locations, we have to solve an eigenvalue problem (11) of size N_d, the number of disturbance sources. For example, the two-dimensional beam truss in Fig. 1 has 56 elements. If we assume that every bar is also a disturbance source, then the size of the eigenvalue problem is $N_d = 56$. On the other hand, we know that there are only $N_p = 12$ CDOF, which means that there is some redundancy in the description of the distortion v_d in Eq. (1). The aim of this section is to exploit that redundancy to reduce the size of the eigenvalue problem.

First we observe that when there are more disturbance sources than CDOF, that is, when $N_d > N_p$, there can be, at most, N_p independent distortion vectors. In this case we can partition the disturbance space into two orthogonal subspaces: $\bar{d}_{\parallel}$, the column space of D, which causes the distortion, and $\bar{d}_{\perp}$, the null space of D, which yields no distortion. These subspaces are orthogonal by virtue of the fundamental theorem of linear algebra[14] and can be found through a singular value decomposition (SVD)[15] of the disturbance influence matrix D. Recall that D is an $N_p \times N_d$ matrix and in our case $N_d > N_p$. Let

$$U[\Sigma_d \, O]V^T = D \tag{13}$$

be the SVD of D, where Σ_d is an $N_p \times N_p$ diagonal matrix of the singular values σ_i of D in descending order, $U(N_p \times N_p)$ and $V(N_d \times N_d)$ are orthonormal matrices, and O is an $N_p \times (N_d - N_p)$ null matrix. If D is of full rank, then all N_p entries of Σ_d are greater than zero. If D is not full rank and has, say, rank r, then only the first r σ_i are different from zero, the remaining ones being null.

We now partition V in the following way. The first N_p columns are defined as $V_{\parallel}$ and the rest as $V_{\perp}$. Equation (13) becomes

$$D = U\begin{bmatrix}\Sigma_d V_{\parallel}^T & O & V_{\perp}^T\end{bmatrix} = U\Sigma_d V_{\parallel}^T \tag{14}$$

From Eq. (14) it is clear that $V_{\parallel}$ is a base of $\bar{d}_{\parallel}$, the part of the disturbance that causes the distortion, whereas $V_{\perp}$ is a base of the null space of D, $\bar{d}_{\perp}$. Substituting Eq. (14) into Eq. (1) yields

$$v_d = U\Sigma_d V_{\parallel}^T d = U\Sigma_d d_{\parallel} = D_{\parallel} d_{\parallel} \tag{15}$$

where $d_{\parallel} = V_{\parallel}^T d$ and $D_{\parallel} = U\Sigma_d$. Matrix $D_{\parallel}$ is the $N_p \times N_p$ reduced disturbance matrix, and the disturbance vector $\bar{d}_{\parallel}$ has only N_p components. Since $d_{\perp}$ does not generate distortion, we can change the constraint in Eq. (5) to include only $d_{\parallel}$. Hence, the size of the eigenvalue problem, Eq. (11), reduces to N_p. This reduction is exact and with no loss of information. Although SVD is an expensive computation, it is done only once, at the preprocessing stage. By following this procedure the overall number of computations is markedly reduced.

In the same vein, if some of the rigid-body constraints are on the CDOF only, say, N_{pc}, the problem size can be further reduced since the last N_{pc} singular values in Σ_d are zeros. This is because rigid-body constraints reduce the number of independent displacements at the CDOF. In that case the base of $d_{\parallel}$ will be the first $N_p - N_{pc}$ columns of V, using the same arguments as before, thus reducing the problem size to $N_p - N_{pc}$. Note that since $N_p - N_{pc}$ is the number of independent distortion modes, this is also the smallest number of actuators required to reduce the error to zero.

Consider again the example in Fig. 1. The number of elements is 56, which is the original problem size. There are $N_p = 12$ CDOF, which reduces the problem size from 56 to 12. There are also $N_{pc} = 2$ rigid-body constraints on the CDOF, y translation and rotation. Consequently we are left with an eigenvalue problem of size 10; furthermore, 10 actuators (at least) can fully control the CDOF.

Heuristic Search Techniques

Now that we have a criterion, J^*, and we know how to calculate its value efficiently, we are left with the problem of finding the optimal configuration, that is, the locations of actuators that will achieve minimum J^*. This is not an easy problem to solve since for a truss consisting of M elements that are all candidates for the N_c control elements, there are, in principle, $M!/N_c!(M - N_c)!$ possible configurations. For small structures bearing a small number of active elements, this problem is manageable through exhaustive search. In the two-dimensional truss example (Fig. 1) with $M = 56$ members, the number of combinations for 4 actuators is 367,290. With nine actuators, however, this number jumps to 7.58×10^9. Large space structures may have thousands of elements and hundreds of actuators and consequently an inordinate number of combinations. Currently it is not computationally feasible to check all possible combinations. Clearly, we have to be more modest in our aspirations.

The problem described earlier belongs to the domain of integer programming or combinatorial optimization. Since getting a globally optimal solution, for this type of problem, requires a prohibitive amount of computation time, we are compelled to use approximation algorithms or heuritics. The search techniques employed in this paper are variants of sequential search or iterative improvement algorithms. In addition to a configuration and a cost function, an iterative improvement algorithm requires a generation mechanism. The generation mechanism is a recipe of how to generate a new configuration from a current one. Note that the generation mechanism implicitly defines the neighborhood of a configuration. When referring to a local optimum it is with respect to that neighborhood. If the generation mechanism is such that it changes only one actuator when moving to a new configuration and we find an optimal configuration, that configuration is optimal with respect to all configurations that differ from the current one by one actuator. It may not be optimal for configurations that differ by two actuators or more.

One increasingly popular Monte Carlo type optimization method is an implementation of the Metropolis[16] algorithm for simulating the equilibrium configuration of a collection of atoms at a given temperature. It is known as SA.[7,8] Following Kirkpatrick et al.,[7] four ingredients are needed in an annealing algorithm: a) a description of the configuration, b) a random generator of transitions or rearrangements of elements in a configuration, c) a quantitative objective function, and d) a cooling schedule giving for ever decreasing temperatures the number of transitions that the system is allowed to perform at every temperature level.

In our case a configuration (a) is described by a set of N_c actuators, one in each location, and noted as I. The objective function (c) is J^*. We perform a transition (b) by replacing one actuator in the current configuration I_0 with a nearby actuator, currently outside I_0, thus obtaining the new configuration I_1. This is done in the following process: A location in I_0, where an actuator will be changed, is selected at random. Let s be the number of the actuator at that location. The actuator number t, of the actuator that is poised to enter the configuration at that location and replace actuator s, is obtained by $t = s \pm \delta$. The + or − is selected at random, and δ is a random generated integer, such that $1 \leq \delta \leq \delta_n$, where δ_n is a predetermined integer that increases or decreases with the size of the problem.

The criterion whether the configuration I_1 is accepted or not depends heavily on the performance index J^* for the new configuration. This is done by the following Metropolis type algorithm. Let ΔJ^* be the increase of J^* when I_1 replaces I_0. If $\Delta J^* < 0$, we have a better response and the new configuration is accepted. If $\Delta J^* > 0$, we have a worse configuration but it may still be accepted. We are here at the crux of SA. Configuration I_1 is accepted if $\exp(-\Delta J^*/cT) > r$, where r is a [0, 1] random number. That is the Metropolis criterion.[16] The probability for accepting worse configuration increases as ΔJ^* becomes smaller. If configuration I_1 is accepted, it becomes I_0 or else I_0 remains unchanged. This concludes the steps for executing a transition.

Finally, we have to determine the cooling schedule (d), that is, the various temperature levels and the time that will be spent at every level. The process starts at a normalized temperature of 1 that is iteratively reduced $T^{(k+1)} \leftarrow T^{(k)}/1.1$ to the freezing point. It was found empirically that for all practical purposes the freezing point corresponds to 0.002, and hence $1 \geq T \geq 0.002$. At every temperature level the system makes transitions a number of times between new configurations according to the algorithm presented earlier. It is clear that as T is reduced, it becomes increasingly difficult to have worsening configurations accepted. To emulate a slow cooling process the number of configuration transitions at every temperature level is given by $f^{(k+1)} = f^{(k)} + \Delta f$ where $f^{(k)}$ is the number of transitions made at iteration k, and Δf is a positive integer defining the increment in transitions between iteration k and $k+1$. Both $f^{(0)}$ and Δf depend linearly on the number of members and the number of actuators. That concludes the definition of the cooling schedule and the annealing process.

Note that to have an adaptive SA algorithm, that is, a problem-size independent procedure, the values of c, Boltzmann's constant in an actual annealing process, and T, the current temperature, are determined as follows. Parameter c acts as a normalizing factor for ΔJ^* and is selected as the limit of J^* for N_c actuators. The temperature T is then normalized and varies between 1 (at the start of the annealing process) and 0.

ESPS is an iterative improvement method that in each cycle performs an exhaustive search for the best configuration that differs from the current one by one actuator only. To find that configuration the algorithm replaces the actuator in the first location of the current configuration with all of the available actuators and finds the best criterion value J_1^* and the corresponding configuration I_1. This is the best value that can be obtained by replacing the first actuator. This step is repeated for all locations, yielding a set of performance values J_i^*, $i = 1, \ldots, N_c$, and a corresponding set of configurations I_i, $i = 1, \ldots, N_c$. The configuration I_i having $\min(J_i^*)$ is the base for the next cycle. The optimization continues until there is no improvement. In one cycle ESPS checks all available actuators in each location, but only one actuator is changed. This, to some extent, is a wasteful procedure since at the start of the optimization the configuration is usually far from optimal and much of the first cycles is dedicated to approaching the optimal neighborhood, one actuator at a time.

To reduce this effect we propose an algorithm that is closely related to ESPS. At every cycle ESPS goes over all locations and in each location tries all available actuators. In the proposed method the cycle starts the same way as in ESPS by finding J_1^* and I_1. Now instead of reverting to the original configuration, the algorithm uses I_1 as a starting point for optimizing the second location and I_2 for the third until the N_cth location. If in one of the steps the original actuator is the best, it will not be replaced. The algorithm stops when after one cycle no actuator has been relocated. In this method the exhaustive search is actuator-wise, or SLIM. In this algorithm more than one actuator may be changed in one cycle, and this will happen mostly in the starting phase. As the design proceeds many actuators will find themselves in optimal positions and will therefore not be replaced.

Lower Limit on the Criterion Value

When using heuristic search techniques there is no guarantee that the configuration thus obtained is optimal. Moreover these techniques do not provide any definite clue as to how good the solution is. It would therefore be very helpful to have advance information on the lower limit of the optimal performance measure. To find the limit we assume, as in Ref. 13, that we have ideal actuators, that is, actuators that can produce any shape we want. In mathematical language, a group of n such ideal actuators can span any subspace with dimension n of the space of the disturbance influence matrix $\boldsymbol{D}$. Equation (15) can be rewritten in the following way:

$$v_d = \boldsymbol{u}_1\sigma_1 d_{\|1} + \boldsymbol{u}_2\sigma_2 d_{\|2} + \cdots + \boldsymbol{u}_{N_p}\sigma_{N_p} d_{\|N_p} \tag{16}$$

where the various $\boldsymbol{u}_i$ are the columns of $\boldsymbol{U}$. Recalling that $\boldsymbol{d}_\|^T \boldsymbol{d} = 1$ and that the various $\boldsymbol{u}_i$ are orthonormal, one can show that without control the largest rms of $\boldsymbol{v}_d$ is σ_1. Indeed,

$$\boldsymbol{v}_d^T \boldsymbol{v}_d = \boldsymbol{d}_\|^T \Sigma_d \boldsymbol{U}^T \boldsymbol{U} \Sigma_d \boldsymbol{d}_\| = \sigma_1^2 d_{\|1}^2 + \sigma_2^2 d_{\|2}^2 + \cdots + \sigma_{N_p}^2 d_{\|N_p}^2 \tag{17}$$

and since $\sigma_i > \sigma_{i-1}$ and $\Sigma_i d_{\|_i}^2 = 1$, it is clear that the largest value of the rms, without control, is obtained for $d_1 = 1$ and $d_{i\neq 1} = 0$. To obtain the limit for one actuator we assume an ideal actuator that can generate the shape of $\boldsymbol{u}_1$. Consequently the control can eliminate the first term in Eq. (17) and the largest size of $\boldsymbol{v}_d$ is now σ_2. In the general case, let N_c be the number of actuators. What shapes should we choose for the ideal actuators to minimize the size of $\boldsymbol{e}$? We propose to use the first N_c $\boldsymbol{u}_i$ in Eq. (17); that is, we select actuator shapes that span the subspace defined by the first N_c columns of $\boldsymbol{U}$. Employing these ideal actuators as control agents will eliminate the first N_c singular values in Eq. (17), and the worst case distortion magnitude will be the $(N_c + 1)$ singular value. This is the best worst case distortion for that number of actuators and we use it as a lower limit on the value of J^* for N_c actuators.

In addition to assessing the quality of a design the limit can be employed for other purposes. Assuming that the performance of real actuators is close to that of ideal actuators, the limit can be used to decide on the number of actuators needed for a desired performance level. Indeed, consider, for example, the case where the design restricts the magnitude of the worst case rms of the error. One then computes the singular values of $\boldsymbol{D}$ and let us assume that the ith singular value is the closest, from below, to the desired rms. Hence $i - 1$ actuators will be needed to reach the design goal. The limit can also serve as an indication of whether the type of employed actuation is adequate. For example, if the actuators are active truss elements and the best value of J^* achieved by the optimization is far from the limit, perhaps more complex control devices, such as tendons, are required. On the other hand, if the best value of J^* is close to the limit, there is no need for different actuators, since even if we had ideal actuators no substantial improvement could be achieved. As mentioned earlier the limit is also used as a parameter in the adaptive SA algorithm since it constitutes a consistent measure of the size of the solution sought by the optimization.

Numerical Examples

The different procedures described herein have been applied to a two-dimensional beam truss and a three-dimensional hexagonal truss, representative examples of flexible structures. The disturbances where taken as individual element length changes, and therefore the number of disturbance sources N_d is equal to the number of elements. The design goal is to maintain the upper line (two dimensions) or surface (three dimensions) of the truss straight. In the following, the criterion value is the worst case rms of the error without concern for the control forces. Hence the weight matrices in Eq. (4) are $\boldsymbol{P} = \boldsymbol{I}$ and $\boldsymbol{W} = \boldsymbol{O}$. Since J^* is the square of the rms, we will use $e^* = (J^*)^{0.5}$ as a normalized rms. For instance, if the disturbance in any bar is limited by, say, 1 mm, a value of $e^* = 6$ (three actuators in Fig. 3) has the meaning that the worst rms of the error is 6 mm. In a different wording, e^* can be construed as a nondimensional amplification factor of the disturbance. The computations were performed on a personal computer using Matlab.[17]

Note that unlike ESPS and SLIM the number of function calls of an SA search is predetermined by the parameters of the cooling schedule. It is important to emphasize at this point that SA used herein retains the best configuration ever generated during the entire process. That is, if during some of the stages of the optimization a minimum J^* was replaced by the Metropolis algorithm with a less efficient design, that minimum J^* is kept in background (far from the annealing transitions). It will be replaced only by a lower J^* configuration. Eventually, this lowest J^* configuration will be used as optimal solution.

ESPS was used with no special modification. The SLIM algorithm was tried with two versions. The first version is exactly as described earlier. In the second version, SLIM(C), if in one location an actuator was changed, that actuator would not be selected in any of the other locations, in that cycle. The reason for this modification was to test the sensitivity of SLIM to small variations of its strategy. As will be seen, SLIM and SLIM(C) behave in a very similar manner. Since the three methods use random data, all of the tested examples were run 25 times and the presented data are average values.

The first tested structure is a two-dimensional truss beam composed of 11 identical bays spanning 27.5 m as shown in Fig. 1. The structure has 24 nodes with an x and y degrees-of-freedom (DOF) at every node giving 48 DOF. It is composed of 56 uniform elements (constant EA), and in principle any one of the elements is a candidate for active control. The set of CDOF are the y displacements of the upper line of the truss (Fig. 1). The purpose of the control is to maintain the upper line (the line of sight) straight; the number of CDOF is thus $N_p = 12$. Since the CDOF possess two rigid-body modes, we have $N_{pc} = 2$, and the problem size here is 10. The optimal actuator placements were found for one up to nine actuators using the search techniques discussed earlier.

We begin with comparing the number of function calls made by each algorithm (Fig. 2). This number is the average of 25 trials. Significantly, for the first actuator SA has about four times more function calls than the other algorithms and for two actuators almost twice more. This is because exhaustive search is relatively easy when the number of actuators is small. For one actuator, for instance, there are only 56 possibilities. The SLIM algorithms have very close

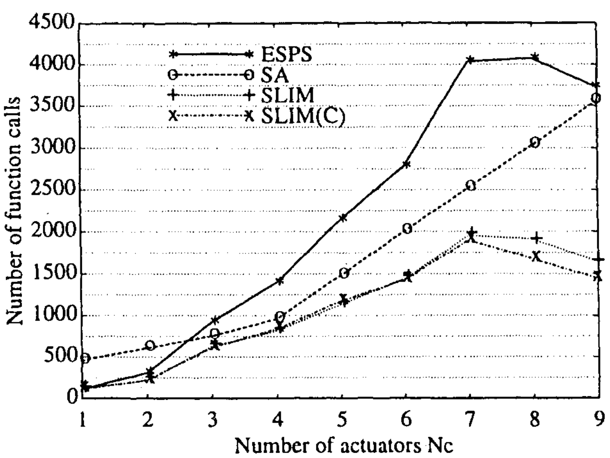

Fig. 2 Average number of function calls for beam truss.

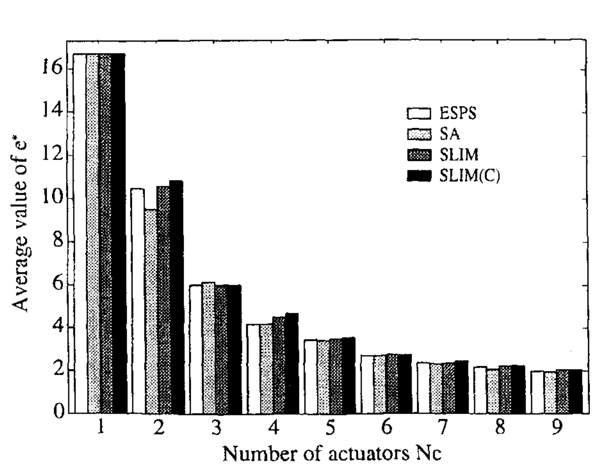

Fig. 3 Average value of e^* for truss beam.

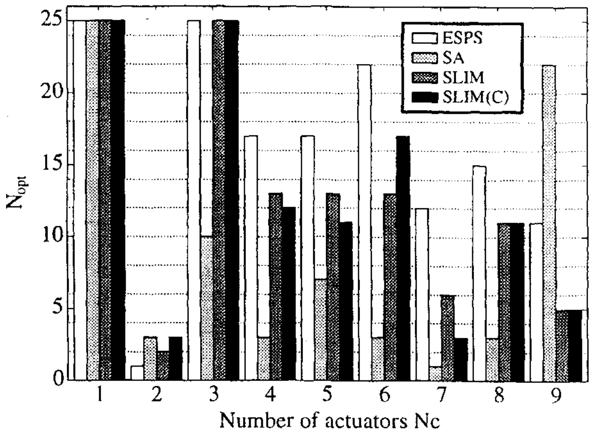

Fig. 4 Number of times each algorithm reached an optimal configuration for the beam truss.

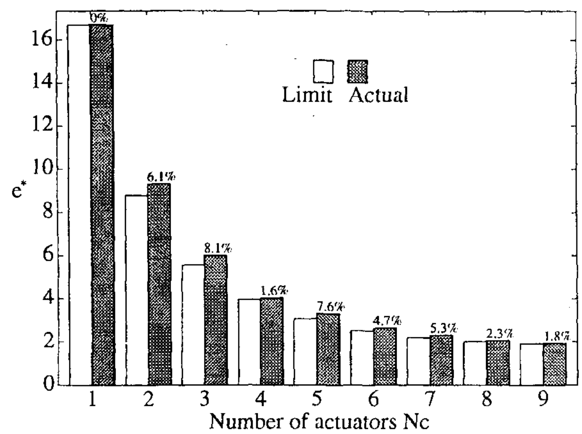

Fig. 5 Deviation of optimal e^* from limit for beam truss.

results, and they have, in general, a better performance than SA and ESPS. For more than four actuators ESPS requires about twice the computation effort of the SLIM algorithms. Up to the seventh actuator there is a steady growth in the number of function calls for all algorithms, and then the SLIM and ESPS algorithms level off, whereas SA continues to grow.

Next we compare the average value of e^* achieved by each of the algorithms (Fig. 3). It is seen that in most cases SA yields better average values and the second best is usually ESPS. When dealing with large problems one cannot hope to find the optimal solution, and consequently the average value is of importance. Note that for this structure all of the algorithms found the same optimum. We have therefore plotted in (Fig. 4) the number of times each algorithm reached that optimal value. For most cases ESPS hit the target the largest number of times, followed closely by the SLIM algorithms and trailed by SA, except for the ninth actuator where it got the best results. An interesting point is to check the validity of the limit and the effectiveness of actuators embedded in truss elements. For that purpose we have plotted the ideal limit and the optimal values achieved in the optimization. It is seen (Fig. 5) that the optimal values are within a 8% range of the limit.

The second structure is a three-dimensional hexagonal truss,[18] with 102 elements (numbered) in Fig. 6. The rods belonging to the upper surface are plotted with solid lines, those of the lower surface with dash-dot lines, and the connecting rods with dotted line. Excluding the rigid-body modes, there are 87 DOF. The CDOF are at the upper surface node displacements in the z direction. There are $N_p = 19$ CDOF and $N_{pc} = 3$ rigid-body modes associated with them (z translation and xz and yz rotations), and therefore the eigenvalue problem is of size 16. Here again we executed each algorithm 25 times, for 1 up to 15 actuators. The number of function calls, for each actuator, in this case (Fig. 7) is almost twice as large as those of the beam truss. This could be foreseen for SA as the candidate locations grew from 56 to 102; however, it is rather unexpected for

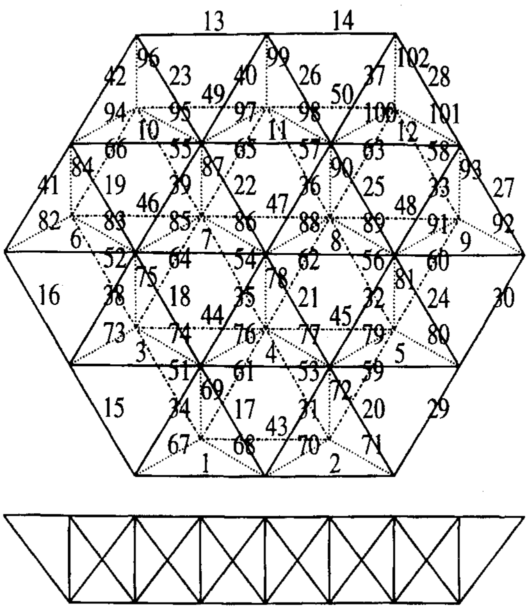

Fig. 6 Antenna truss.

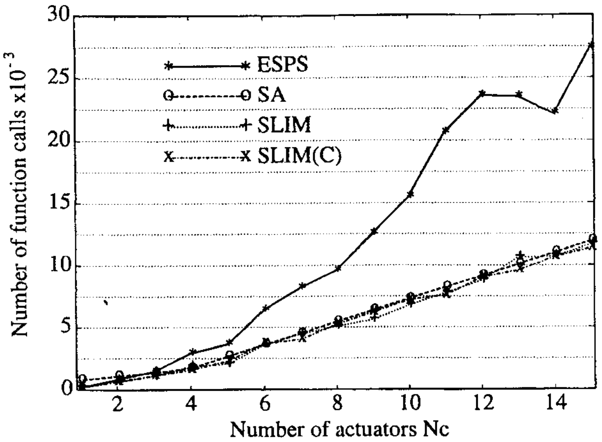

Fig. 7 Average number of function calls for antenna truss.

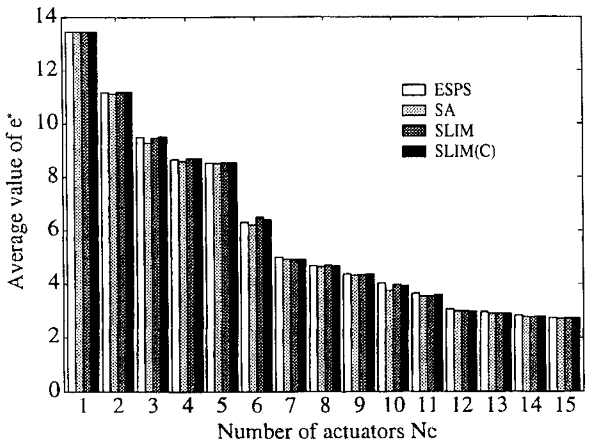

Fig. 8 Average value of e^* for antenna truss.

ESPS and SLIM. The SLIM algorithms and SA have almost the same results, whereas ESPS requires close to twice their number of function calls. The number of function calls grows almost linearly in terms of N_c except for a local drop with ESPS for the 13th and 14th actuators.

Comparing the average value of e^* attained by the different methods (Fig. 8), all techniques have similar results. Here too, as in the beam truss, SA has better averages in most cases, but it is

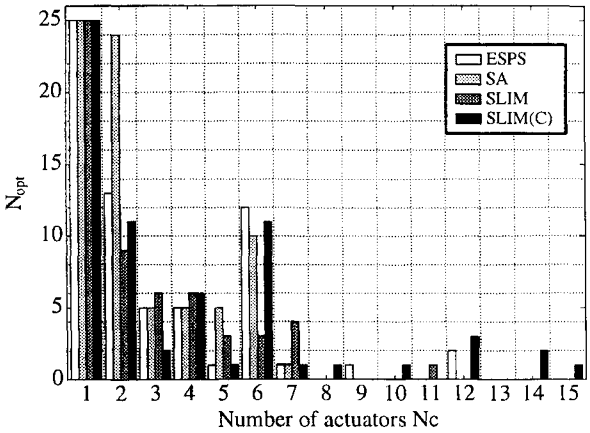

Fig. 9 Number of times each algorithm reached an optimal configuration for antenna truss.

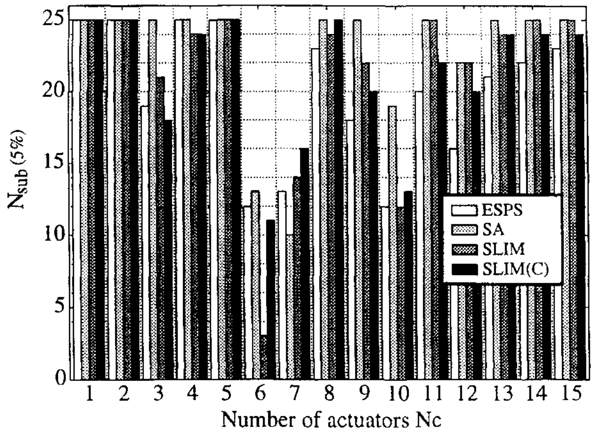

Fig. 10 Number of times each algorithm reached within 5% of optimal e^* for antenna truss.

less noticeable. When comparing the number of times each method obtained an optimal configuration (Fig. 9), the situation differs from the previous case. For the first actuator all of the techniques have good results, since there are only 102 possibilities. From two up to seven actuators there is a gradual decrease, except in the case of six actuators where there is an increase. For more than seven actuators achieving an optimal configuration is very rare. Beyond seven actuators only one method would find an optimal configuration, and usually only once. For example, SA did not achieve any optimal configuration, ESPS got it once for 9 actuators and twice for 12, and for 13 actuators no method achieved the optimal configuration. Note that the optimal configurations were found during additional trials performed beyond the standard 25. This is a dire situation since no method succeeded in locating the optimum. However, scrutiny of the results will mitigate this statement. In Fig. 10 we give the number of times each method came within 5% of the optimal results. These are evidently encouraging results.

Improved Control

Checking the optimal results of the antenna against the ideal limit (Fig. 11) shows that there is, in general, good agreement except maybe for the case of three actuators, which exhibits a 8.9% deviation. As a result we can tentatively conclude that, for the three-dimensional antenna truss, the performance of actual active elements can get close to that of the ideal ones. This is also true for the beam truss. Two conclusions can be drawn. In a first instance, in most cases there is no need for more complex actuators. Second, the limit can be used as an estimation for the number of actuators that will be needed for a structure in a given level of performance.

The relatively large deviation in the case of three actuators of the antenna truss could be the result of a faulty optimization, and there may be a better solution in store. If this is the case, the search should be resumed. However, this is not probable since for three

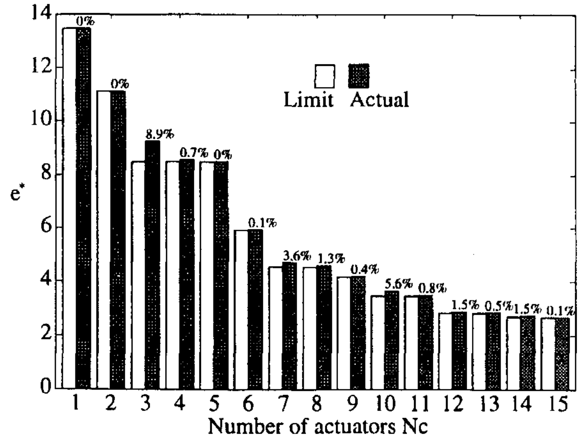

Fig. 11 Deviation of optimal e^* from limit for antenna truss.

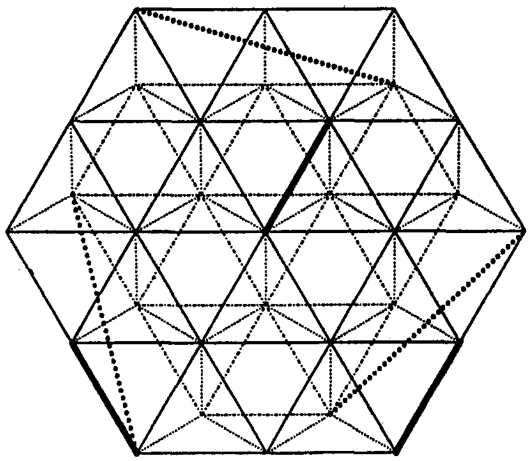

Fig. 12 Optimal configurations for three active truss elements and three tendons: ——, active members; ......, tendons.

actuators the number of combinations is relatively small (approximately 172,000). Therefore we must assume that the type of actuation used is not adequate enough. To check this assumption we tried tendon actuation, that is, actuators that can be attached to any two nodes of the structure. For the case at hand there are 465 available locations, instead of the original 102. The optimal locations of the active elements and tendons are shown in Fig. 12, and indeed this configuration is better (3.26% deviation from the limit as compared with 8.9%). This is an example of how the limit can be used as a two-way alley. If the deviation of the optimization results from the ideal limit is substantial, the search should be resumed. If we are confident that the current configuration is optimal for that type of actuation, we may want to use a different type of actuation or increase the number of actuators.

Conclusions

This paper implements the worst case distortion criterion on two- and three-dimensional trusses, representative of flexible structures. The main problem is the size of the optimization space, which precludes exhaustive search, and heuristic techniques seem to be the preferred solution method. It is shown that heuristics have satisfactory performance in two important categories. First, they obtain results close to the theoretical limit, and second, the number of function calls grows linearly with the problem size and not combinatorially as expected in this type of problems. All methods behave very similarly. From a computational point of view, SA and SLIM, the method presented herein, exhibit close performances, whereas ESPS requires almost twice their computation effort. In most cases the method with minimum average value of J^* is SA.

The limit introduced here is a very handy design tool. It can be used before the optimization to decide on the number of actuators. In the case of SA it is also used in the optimization process as one of the parameters. In addition, it can be employed after the optimization to check the results. If the deviation of the actual result from the limit is large, we may want to resume the optimization. If we are confident with the results, one can try other types of actuation, as was done with the antenna with three actuators, or alternatively increase the number of actuators.

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